

Selection of Optimal Portfolio: A Conceptual Study

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Abstract:

The Entire process of portfolio selection begins with portfolio construction, which can be simply viewed as a matter of selecting securities to include in a portfolio and determining appropriate weighting to each of the securities in the portfolio. This study briefly describes how an investor selects a portfolio from a set of efficient portfolios based on his/her level of risk aversion.

Here among other useful concepts Sharp model and Markowitz's portfolio theory have been used. These two models are very much useful to select the optimal portfolio, which is also the most efficient one.

The Sharp Model is used to construct the portfolio with the securities, which have the most expected return with a desired risk. And the Markowitz Model indicates the proper goal of portfolio construction should be to generate a portfolio that provides the highest return for a given level of risk. A portfolio having such characteristics is called an efficient portfolio.

After the construction of efficient portfolio, an investor can choose his/her portfolio from the set of efficient portfolios. This selected portfolio from the set of efficient portfolios is called optimal portfolio, which has the best risk return combination.

Key words: Portfolio, securities, optimal, Investment,

Introduction:

Investment is one of the most important concept in economics. Growth and prosperity of an economy to a large extent

depends upon growth in investment. Savings are the starting point of investment. Household sector saves a portion of his income on regular basis. This fund is channelized to an investment opportunity to earn some extra money. This extra income compensates the fall in money value. However, investment in a source is not in real sense an effective investment. It can be considered as money investment. This money is pass on to business sector. They use this money to buy different factors of production for investing in business production activities. It will increase aggregate production of the economy or national income.

Since saved money is scarce, it is required to be used effectively. So household sector will select an investment option where return is optimum. However, investment environment is risky. Return and risk are positively related. So risk increases with the increase in return. Every investor has a risk bearing capacity. Basically, they are risk averter. So, they try to earn a reasonable return subject to a minimum risk.

Basic question here is the method of measuring risk and return. Markowitz for the first time has highlighted on the method of measuring risk and return. He has also established the existence of positive relation between return and risk. He has analysed different components of risk on the basis of sources of risk. In his opinion, risk can be of two types viz. Systematic and unsystematic. Systematic risk arises from different factors lying outside the business. They are common factors affecting all business sectors of the economy. Inflation, political unrest, natural calamities etc. Unsystematic risk arises from factors inside the business. Labour unrest, management inefficiency, obsolete machine, low productivity etc. are causing unsystematic risk. They can be reduced by taking suitable action.

Markowitz has stated that unsystematic risk can be reduced to almost zero. It requires investment in diversified portfolio of securities. Investment in one company's share is very risky. It gets distributed if investment is made in more than one securities. This collection of invested securities is known as portfolio. But nature of share should not be the same. Proper diversification of portfolio can yield optimum result. He has

calculated beta value of each share. Similar type of beta value should not be used in portfolio. More variation in beta value will mean better portfolio formed. Markowitz has established that well diversified portfolio can eliminate unsystematic risk to almost zero.

Markowitz concept has been analysed by other authors like Sharpe, Lintner and others to develop Capital Asset Pricing Model (CAPM). However, limitation of his model is calculation of unknowns. Number of unknowns increases in multiplicative manner with each unit addition of shares in portfolio. In order to avoid this complication Sharp has developed an alternative method of developing portfolio. It reduces calculation of unknowns to a large extent.

Literature review:

H. Markowitz (1952) has developed the method of calculating risk and return. In risky environment expected mean value indicates expected return or yield. Standard deviation is used for risk. He has suggested cofactor for deciding the best investment option. It is ratio of return per unit of risk. It is known as mean-variance (MV) theory. Further he has suggested a diversified portfolio of stocks to minimise risk subject to a best possible return. Lee, Cheng and Chang (2010) in his research work has supported the idea of diversified stock as suggested by Markowitz. It will reduce company specific unsystematic risk to almost zero. (Goetz man et al.2014) has shown that unsystematic risk can be reduced to almost zero in his research work. Based on MV model, Sharpe (1964) and Lintner (1965) have developed Capital Asset Pricing Model (CAPM). This model has added risk free bond with optimal risky equity model of Markowitz to form capital market line (CML). An investor can move along this line to improve his satisfaction level of investment. Also, the idea of Markowitz model has been extended to develop a Security market line (SML). It can be used to decide under and overpriced stock.

Esmaelli, Souri and Mirolohi (2020) has shown that 1st and 2nd moment becomes ineffective if CAPM model fails normality test. They suggested use of 3rd and 4th moment for portfolio optimization in this situation. CAPM model has

been suggested by Rosi (2016) due to its simplicity provided complete information are available. Otherwise, effectiveness of this model will be reduced. Assumptions of this CAPM model are making it ineffective as stated by Yang (2021). Maiti and Balakrishna (2020) have stated the usefulness of debt-to-equity ratio in explaining stock return. Fama and French have introduced third factor along with return and risk. Afterwards Five factors model has been suggested by Jain and Singla (2021). They have included leverage and liquidity in CAPM model for enhancing its effectiveness.

Marhowitz Model (1952) of optimum portfolio is single period quadratic programming problem. So, when number of stocks included in portfolio is on the higher side, estimating covariance matrix becomes very difficult. For overcoming this limitation, Sharp, (1963) has formulated a single index model known as 'portfolio optimization'. It has reduced calculations from $[N(N-1)/2]$ to $3N+2$. Also, this model has simplified the calculation of beta value (Mondal, 2013)). Konno & Yamazaki (1991) proposed mean-absolute deviation model (MAD) as a substitute of MV model. It can yield same result as MV model with normality distribution.

Sharp Index model was formed by Dr. Aloysius Edward & Jagadish, (2020). It shows that only 5 top private banks are considered in optimum portfolio. required here for portfolio optimization. Yang, (2021) made a comparative study CAPM and Sharp ratios. He observed that two models are simple and easy to understand. But they have made too many simplifications and leads to inaccurate results.

Next considered number of shares in optimum portfolio. Mbithi, (2014) carried out a research work in Nairobi Stock Exchange. It shows number of shares varies from 18 to 22 shares in optimum portfolio. Lee, Cheng & Chang (2016) observed that CAPM model is very suitable in Malaysian stock exchange as it can reduce the unsystematic risk to almost zero. So far as research work in India is concerned, Chauhan (2014) constructed a portfolio using top 10 stocks of Nifty. He observed that Sharp optimum portfolio is simplest method of calculating optimum portfolio. Nalini, (2014) considered 15 stocks of various sectors from BSE sensx.

It was observed that risk can be reduced by diversifying portfolio. Only 4 stocks are selected for inclusion in optimum portfolio. Gopalakrishnan (2017) in his study of 13 actively traded scripts of NSE IT index. He concluded that a significant relation exists between IT Index and S&P Nifty 50 index. Tanuja & Srivastava (2017) has formed a Sharp index model from Nifty 50 stocks. The method of constructing optimum portfolio has been explained here.

Role of computational intelligence a sub system of artificial intelligence, by using as taken a prominent part today in the analysis of Risk and Return. Portfolio optimising problem can be solved by using this technology. Alireza Nazemi and Narges Tahamasbi (2024) has shown it in his research work. In April 2022, Sulaiman Olanerewaju Adebisi have published an article based on Nigerian exchange market. They have used computational intelligence technique on historical price data set of listed firms to develop optimum risk-return portfolio. Alirezo Goli, Hasan Khademi Zare and others in 2019, has conducted a case study on hybrid artificial intelligence and robust optimisation problem of multi objective in nature. Another paper published by Rodolfo C. Cante and R.C.Brasileiro deals with market information system. Many scientific papers published in last decade to investigate use of CI method to financial market. They have focused on review of literature on this topic and discussed on main challenges and open problems in this scientific field. Akashdeep Das, Tinny Chowdhury and others have compared performances of techniques like Particle Swarm Optimisation (PSO), Genetic Algorithm (GO), Dynamic Programming (DP), Differential Evolutionary Algorithm (DEO) in optimising portfolio. They have considered Nifty 50 daily price data from March to May 2023 for this purpose. Results show that PSO is best to increase return at constant risk. Abhishek Gunjan and Siddhartha Bhattacharyya (2023), has dealt with portfolio selection problem in a volatile market. They applied different classical, statistical and intelligent approaches in portfolio optimisation problems. They have also showed how application of different machine learning and artificial intelligence can help portfolio managers to take right decision in allotment of portfolio optimisation. Ankit Dangi (2013),

has considered the role of using different weights in portfolio optimisation. He has formed a basket of optimum portfolios using different weights. Then a global optimum portfolio has been selected using computational intelligence techniques

Objective of study:

The Entire process of portfolio selection begins with portfolio construction, which can be simply viewed as a matter of selecting securities to include in a portfolio and determining appropriate weighting to each of the securities in the portfolio.

The Sharp Model is used to construct the portfolio with the securities, which have the most expected return with a desired risk. And the Markowitz Model indicates the proper goal of portfolio construction should be to generate a portfolio that provides the highest return for a given level of risk. A portfolio having such characteristics is called an efficient portfolio.

After the construction of efficient portfolio, an investor can choose his/her portfolio from the set of efficient portfolios. This selected portfolio from the set of efficient portfolios is called optimal portfolio, which has the best risk return combination.

Objective of this study is to construct portfolio using CAPM and SHARP method by using 50 shares used in Nifty 50 index of National Stock Exchange.

Data used:

Research work is based on primary and secondary data. No primary data is required here. We have done the calculations on the basis of daily price movements of 50 shares included in NIFTY50 Index for the financial year 2021-22. Closing prices are considered.

Methodology:

Since investment decisions are taken in a risky situation where more than one outcome are possible. But probability of outcomes is predictable. As per Markowitz, expected return is the yield value of a stock. Expected standard deviation is risk. Formula used for them are –

$$E(R) = P_1 R_1 + P_2 R_2 + \dots + P_n R_n = \sum_{i=1}^n P_i R_i$$

$$\text{Risk} = \sigma_X = \left[P_1 (R_1 - \bar{R}_X)^2 + P_2 (R_2 - \bar{R}_X)^2 + \dots + P_n (R_n - \bar{R}_X)^2 \right]^{\frac{1}{2}}$$

In the formula P_i indicates probability of occurrence of return R_i . As we are using closing stock price the following formulas will be used that is applicable for portfolio of two securities.

$$\text{Yield} = E(P) = \bar{R}_P = W_1 \bar{R}_1 + (1 - W_1) \bar{R}_2$$

$$\text{Risk} = \sigma_P = \left[W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + 2W_1 W_2 \sigma_{12} \right]^{\frac{1}{2}}$$

In the formula P_i indicates probability of occurrence of return R_i . As we are using closing stock price the following formulas will be used that is applicable for portfolio of two securities.

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Where,

$\bar{R}_i$ = expected return of stock 'i'

σ_i = risk of security 'i'

σ_{ik} = covariance of security 'i' and 'k'

w_i = weight factor of security 'i'

Instead of 2 securities when more numbers are added, then the formulas are-

$$\text{Portfolio return} = w_1 \bar{R}_1 + w_2 \bar{R}_2 + \dots + w_n \bar{R}_n = \sum_{i=1}^n w_i \bar{R}_i$$

$$\text{Risk} = \sigma_P = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij}$$

$$= \left[\frac{1}{n^2} \sum_{\substack{i,j=1 \\ i=j}}^n \sigma_{ij} + \frac{1}{n^2} \sum_{\substack{i,j=1 \\ i \neq j}}^n \sigma_{ij} \right]^{\frac{1}{2}}$$

Portfolio risk formula of n securities has two parts. Let us consider the effect of increasing securities on First part.

$$\frac{1}{n^2} \sum_{\substack{i,j=1 \\ i=j}}^n \sigma_{ij} = \frac{1}{n^2} \sum_{i=1}^n L = \frac{1}{n^2} \times n = \frac{L}{n}$$

$$\therefore \text{If } n \rightarrow \infty \text{ then } \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

In the above mathematical explanation, first part shows individual security variance. L is considered as the largest variance of a security. It is replaced for all variances of individual securities. Then first part is simplified as L/N . Here N is the number of securities. If it increases in numbers then first part will tend to become. Thus limit of first part of the portfolio risk is zero when N tends towards infinity.

Next we consider the second part of portfolio risk formula for n securities.

$$\frac{1}{n^2} \sum_{\substack{i,j=1 \\ i \neq j}}^n \sigma_{ij} = \frac{1}{n^2} \sum_{i,j=1}^n n(n-1)\sigma_j = \frac{n-1}{n} \sum_{i,j=1}^n \sigma_j = 1 - \frac{1}{n} \sum_{i,j=1}^n \sigma_j$$

$$\therefore \lim_{n \rightarrow \infty} \sum \sigma_{ij} - \lim_{n \rightarrow \infty} \frac{1}{n} \sum \sigma_{ij} = \bar{\sigma}_{ij}$$

It consists of covariance of two different securities. All such covariance of possible for two securities included in portfolio are taken. It is observed that result of this second part tends towards a constant value.

Calculation of portfolio from 50 stock of Nifty50 index requires following steps;

Methodology for beta diversification portfolio:

- Daily price returns are calculated first. It is showing percentage change in current day t in comparison with closing price of previous day. Formula is-

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \times 100$$

Same formula is used to ascertain daily returns for whole year of all 50 stocks in Nifty50 index.

- On the basis of calculated daily returns, mean return, variance and beta values are calculated for shares and market index for the financial years.

- Next is the formation of portfolio. It has been constructed by using risk minimising weight X_i . the formula used for calculating weight is –

$$X_i = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$$

- At first all 50 shares are arranged on the basis of their beta value in descending order. Only two shares one from top and second one from bottom are included in first portfolio. Risk and yield of this portfolio has been estimated.

- Then first portfolio has been considered as a composite share. One more share is added with it to form portfolio 2. This process continued until all 50 shares are included. Thus 49th portfolio is the final portfolio of all 50 stocks and its yield and risk is overall portfolio return and risk.

Methodology for constructing Sharp Optimal portfolio:

Main limitation of the Markowitz model is calculation of unknown's increases in a multiplicative manner. Sharp's single indexed optimum portfolio model (1963) can solve this problem. It is based on following assumptions-

-Expectations of all investors are homogeneous.

-In determining risk and return of each stock, same period's data are considered.

-Existing Economic and Business conditions have major impact on movement of stock price.

-Index with which securities returns are linked has been constructed from the stocks of same market.

Here selection of stocks for inclusion in optimum portfolio is made on the basis of excess beta ratio. It is excess return over risk free return for per unit beta value. This method first identifies stocks included in optimum portfolio by using

cut off rate (c^*). Then weight factors are determined for each selected stock. Portfolio return and risk are finally calculated on the basis of such calculations.

Steps of optimum portfolio formation in sharp model are explained below-

1. Calculation of excess beta value of all stocks are computed by using the formula-

$$= \frac{R_i - R_f}{\beta_i}$$

On the basis of these calculations, shares are ranked in descending order.

2. Next step is the calculation of C_i value of each share. The formula is-

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^N \frac{(R_i - R_f) \beta_i}{\sigma_e^2}}{1 + \sigma_m^2 \sum_{i=1}^N \frac{\beta_i^2}{\sigma_e^2}}$$

3. Now all C_i values will be compared. It will increase initially. Then it will be optimum. After that C_i value will decline. Optimum C value is cut off rate. All shares ranked higher than cut off C value is included in optimum portfolio.

4. After identifying stocks the weight factor of each selected stock is estimated. At first Z value is calculated by the formula-

$$Z = \frac{\beta_i}{\sigma_e^2} \left[\frac{R_i - R_f}{\beta_i} - c^* \right]$$

5. Finally sum of Z values are taken and proportion of each Z value on sum Z value are ascertained. They are taken as weight for inclusion in optimum portfolio. On the basis of these weights all selected equities are included to form the optimum portfolio.

Results:

Data of year 2020-21 are used here to show portfolio construction using CAPM method with Xi weight on the basis of beta diversification.

Portfolio	Stock 1	Stock 2	Xi value	Return	Risk	Cofactor
1	Cipla	Ind Bank	0.233162	0.446953	3.127589	0.142907
2	Portfolio 1	Axis Bank	0.629436	0.401594	2.982658	0.134643
3	Portfolio 2	ICICI Bank	0.510091	0.350578	2.816481	0.124474
4	Portfolio 3	Bajaj Fin.	0.669023	0.363668	2.691896	0.135097
5	Portfolio 4	Hindalco	0.684711	0.427112	2.533683	0.168574
6	Portfolio 5	TATA Motors	0.853227	0.462211	2.493755	0.185347
7	Portfolio 6	Bajaj Finserv	0.710113	0.429012	2.425214	0.176897
8	Portfolio 7	Housing Dev	0.555053	0.333863	2.277649	0.146582
9	Portfolio 8	SBI	0.745745	0.326455	2.226072	0.146651
10	Portfolio 9	M & Mahindra	0.755397	0.361951	2.141996	0.168978
11	Portfolio 10	JSW steel ltd.	0.760169	0.398001	2.089879	0.190442
12	Portfolio 11	HDFC Bank	0.574468	0.339742	1.967274	0.172697
13	Portfolio 12	Maruti Suzuki	0.750264	0.310501	1.901126	0.163325
14	Portfolio 13	TATA Steel	0.885394	0.330502	1.88609	0.175231
15	Portfolio 14	Kotak Mahindra	0.782331	0.2993	1.838603	0.162787

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16	Portfolio 15	Adani Port	0.744246	0.338349	1.738181	0.194657
17	Portfolio 16	Grashim Ind.	0.835466	0.360949	1.712761	0.210741
18	Portfolio 17	BPCL	0.832371	0.329345	1.666008	0.197685
19	Portfolio 18	TITAN	0.82002	0.311654	1.629883	0.191213
20	Portfolio 19	Hero Motor	0.877599	0.305458	1.610288	0.189692
21	Portfolio 20	Reliance Ind.	0.772214	0.298398	1.533994	0.194523
22	Portfolio 21	UPL Ltd	0.933102	0.301308	1.523508	0.197773
23	Portfolio 22	Shree Cement	0.881044	0.295946	1.508858	0.196139
24	Portfolio 23	Bajaj Auto	0.892395	0.291737	1.498175	0.194728
25	Portfolio 24	Eicher Motor	0.998203	0.291112	1.498136	0.194316
26	Portfolio 25	Ultra Tech	0.882384	0.295486	1.48728	0.198676
27	Portfolio 26	ONGC	0.930836	0.289764	1.478222	0.196022
28	Portfolio 27	L & T	0.858669	0.286327	1.458771	0.19628
29	Portfolio 28	IOCL	0.924349	0.270852	1.454112	0.186266
30	Portfolio 29	Coal India Ltd	0.944958	0.255698	1.451234	0.176194
31	Portfolio 30	HCL Techno.	0.842914	0.274283	1.415237	0.193807
32	Portfolio 31	Infosys	0.838216	0.286682	1.387103	0.206677
33	Portfolio 32	SBI Life	0.831691	0.26393	1.355549	0.194703
34	Portfolio 33	HDfC life	0.880387	0.2578	1.342877	0.191976
35	Portfolio 34	TCS	0.839082	0.25936	1.319529	0.196555

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36	Portfolio 35	Tech Mahindra	1.101826	0.25631	1.31096	0.195513
37	Portfolio 36	Bharti Airtel	0.875476	0.238083	1.280813	0.185884
38	Portfolio 37	Wipro	0.78521	0.260124	1.260911	0.206299
39	Portfolio 38	Asian Paint	0.82716	0.250437	1.220809	0.20514
40	Portfolio 39	NTPC	0.913921	0.240426	1.208844	0.198889
41	Portfolio 40	TATA Cons.	0.903781	0.251368	1.195612	0.210242
42	Portfolio 41	Power Grid	0.877326	0.23937	1.174973	0.203724
43	Portfolio 42	Divis Lab.	0.929522	0.242761	1.167356	0.207958
44	Portfolio 43	Sun Pharma	0.930718	0.24297	1.159872	0.20948
45	Portfolio 44	Nestle	0.855599	0.215385	1.138555	0.189174
46	Portfolio 45	Hindustan Uni.	0.874543	0.195653	1.123986	0.17407
47	Portfolio 46	Britania Ind	0.866822	0.189906	1.109379	0.171182
48	Portfolio 47	ITC	0.845971	0.180398	1.070826	0.168467
49	Portfolio 48	Dr Reddys Leb	0.978156	0.180239	1.070106	0.168431

Analysis: The construction of portfolio has been started with Equities of CIPLA and Indus Ind Bank. Indus Ind Bank has highest beta value and CIPLA is with lowest beta. Risk and return of first portfolio are estimated. Then with portfolio 1 AXIS bank with next highest beta value has been combined to form portfolio 2. This process continued up to 49th portfolio when all 50 shares are included. Thus, Final return and risk of 50 shares are 0.180239 percentage and 1.070106 percent. It has cofactor of 0.168431 per unit of risk

TABLE 2: Optimum Portfolio of Sharp method

SERIAL	NAME OF EQUITIES	MEAN RETURN	WEIGHT	RISK
1	Eicher Motors Ltd.	0.383427	0.025752	1.099355
2	Power Grid Corporation of India Ltd.	0.378153	0.250165	3.429078
3	Shree Cement Ltd.	0.372918	0.203109	3.09215
4	Tata Consultancy Services Ltd.	0.323926	0.240868	3.391369
5	GAIL (India) Ltd.	0.322252	0.158221	2.749309
6	JSW Steel Ltd.	0.314916	0.121885	2.749309
	Portfolio Mean	0.347612		
	Portfolio risk		16.17687	
	Cofactor			0.021488

Analysis: In Sharp portfolio of table 2 only 6 shares are included. They have C value below cut of C value 5.900347. All shares lying below Cut off value are excluded from the optimum portfolio. This portfolio has low portfolio return 0.347612% and high risk 16.1787. and low cofactor of 0.021488 per unit of risk.

Evaluation:

1. In CAPM beta diversification portfolio risk is gradually coming down with each addition in share. It is shown in diagram 1 below-

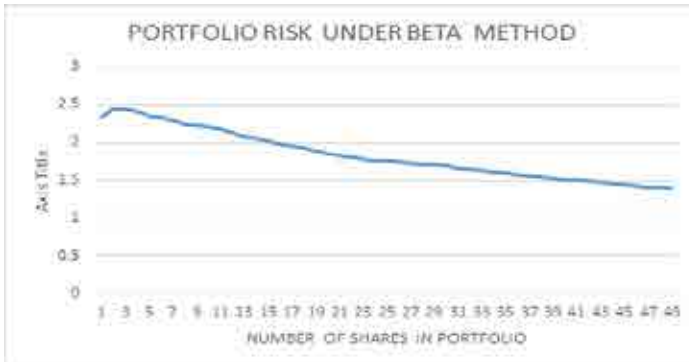


Diagram 1; Portfolio Risk

2. Also, portfolio risk is much less than portfolio risk under SHARP optimum method.

3. Further cofactor is also well above the cofactor of SHARP optimum portfolio.

4. However, number of shares included in portfolio of SHARP is much lower than CAPM optimum portfolio. It is only 6 numbers in comparison with 50 numbers in CAPM portfolio.

Conclusion:

It can be concluded that CAPM beta diversification portfolio is providing more effective return, risk and cofactor. So, investor in Indian economy can go for this method in constructing optimum portfolio provided they have capacity to invest in 50 shares. Small investors with limited investible fund can go for SHARP optimum portfolio. This analysis is based on only one-year data. Same analysis can be extended for at least 5 to 6 years to get more comprehensive conclusion.

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